1. Calculate the eccentricity of the ellipse $6x^2 + 4y^2 = 48$.

   (a) $\frac{2}{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\sqrt{2}$ (e) $\frac{\sqrt{3}}{3}$

2. The Morey family has two children. If we know that at least one of the children is a girl, find the probability that both are girls, given that the probability of getting a boy is equal to that of a girl.

   (a) 1/2 (b) 1/4 (c) 3/4 (d) 1/3 (e) 1

3. Find the inverse, if possible, of $f(x) = \ln(x + \sqrt{x^2 + 1})$.

   (a) $g(x) = \frac{1}{\ln(x + \sqrt{x^2 + 1})}$ (b) $g(x) = e^{(x+\sqrt{x^2+1})}$ (c) $g(x) = e^{2x} - 1$

   (d) $g(x) = \frac{e^{2x} - 1}{2e^x}$ (e) Not possible

4. Which of the following is the negation of the statement, "All good things come to an end"?

   (a) All good things do not come to an end.

   (b) All good things last forever.

   (c) Some good things do not come to an end.

   (d) No good things come to an end.

   (e) Nothing good lasts forever.

5. Suppose $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist. If $\lim_{x\to a}(f(x) + g(x)) = 2$ and $\lim_{x\to a}(f(x) - g(x)) = 1$, find $\lim_{x\to a}(f(x)g(x))$.

   (a) 2 (b) 3 (c) 3/4 (d) 3/2 (e) 1/2

6. Let $f(x)$ be a differentiable function. Let a be a real number with the property that $f'(a) \neq 0$. Then the y-intercept of the tangent line to the graph of f at $x = a$ is

   (a) $f'(a)$ (b) $f(a)$ (c) $a - \frac{f(a)}{f'(a)}$ (d) $-a\ f'(a) + f(a)$ (e) $f'(a) - f(a)$

7. Let $f(x) = \sqrt{4 - x^2}$. Find $f(f(f....f(1)))$ where f is composed with itself 2002 times.

   (a) 1 (b) $\sqrt{3}$ (c) 2 (d) $\frac{1}{\sqrt{3}}$ (e) Cannot be determined

8. Suppose you want to create a cone by cutting a sector from a circle having a radius of $\frac{5}{2}$ meters and folding the remainder of the circle into a cone. The circumference of the base of the cone is 4π meters. What is the measurement of the central angle (in degrees) subtending the sector which needs to be removed?

   (a) 0° (b) $33\frac{1}{3}^\circ$ (c) 36° (d) $66\frac{2}{3}^\circ$ (e) 72°

9. Suppose set A has m elements and set B has n elements where $m > n$. If the set $A \cup B$ has m elements, which of the following statements are true?

   I. $B \subseteq A$ **II.** $A \subseteq B$ **III.** $A \cap B = B$

   (a) I only (b) II only (c) III only (d) Both I and II (e) Both I and III

10. Find the sum of all values of k that satisfy:

$$2^{2k} - 2^{k+4} + 48 = 0$$

   (a) 2 (b) 4 (c) $2 + \dfrac{\ln 2}{\ln 3}$ (d) $2 + \dfrac{\ln 3}{\ln 2}$ (e) $4 + \dfrac{\ln 3}{\ln 2}$

11. Suppose you have an equal number of pennies, nickels and quarters in your hand. The total amount of money in your hand is \$4.65. How many total coins do you have in your hand?

   (a) 10 (b) 15 (c) 30 (d) 45 (e) 60

12. Find all zeros of $f(x) = x^5 + 3x^3 - 64x^2 - 192$.

   (a) $3, -3, 4, -2 + 2\sqrt{3}i, -2 - 2\sqrt{3}i$ (b) $3, 3, 4, -2 + 2\sqrt{3}i, -2 - 2\sqrt{3}i$

   (c) $3i, -3i, 4, -2 + 2\sqrt{3}i, -2 - 2\sqrt{3}i$ (d) $\sqrt{3}i, -\sqrt{3}i, 4, -2 + 2\sqrt{3}i, -2 - 2\sqrt{3}i$

   (e) $3, 4, 8, -1 + \sqrt{3}i, -1 - \sqrt{3}i$

13. Two strips of width m and n, respectively, intersect at an angle α. What is the area of their intersection?

   (a) $mn \sin \alpha$ (b) $\dfrac{mn}{\sin \alpha}$ (c) $\dfrac{mn}{\sin^2 \alpha}$ (d) $\dfrac{mn}{1 - \cos \alpha}$ (e) mn

14. Find the median of the following table of values:

20	10	7	19	5
12	2	13	4	18
8	17	16	9	1
3	6	11	14	15

(a) 10 (b) 12 (c) 11 (d) 10.5 (e) None of the above

15. Simplify $4^{40} \cdot 5^{40} \cdot 4^{50} \cdot 5^{50}$.

(a) 9^{90} (b) 9^{200} (c) 20^{90} (d) 20^{200} (e) 400^{180}

16. The quotient $\dfrac{|e-3|}{|\pi-3|}$ is exactly equal to which of the following?

(a) 1.989638335 (b) $\dfrac{e}{\pi}$ (c) $\dfrac{e+3}{\pi+3}$ (d) $\dfrac{3-e}{\pi-3}$ (e) 0.8652559794

17. If $\log_2(\log_2(x)) = 5$, how many digits are in the base-ten representation for x?

(a) 32 (b) 20 (c) 10 (d) 9 (e) 3

18. For a particular bike, one rotation of the pedals will rotate the wheel of the bike twice. If you make 100 rotations of the pedals, how much farther will the bike travel with tires of diameter $2\ \frac{1}{2}$ feet than it will with tires of diameter 2 feet?

(a) 0 feet (b) 50 feet (c) 100 feet (d) 100π feet (e) 50π feet

19. The expression $\cot 2x + \tan x$ is equal to which of the following?

(a) $\csc x$ (b) $\csc 2x$ (c) $\sec x$ (d) $\sec 2x$ (e) $\sin 3x$

20. Which of the following are true statements?

I. $\dfrac{1}{a+b} = \dfrac{1}{a} + \dfrac{1}{b}$ is not always true, but it is true for some nonzero real numbers a and b.

II. $\sqrt{1+a} = 1 + \sqrt{a}$ is not always true, but it is true for some nonzero real number a.

III. $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ is not always true, but it is true for some real numbers a and b.

(a) I, II and III (b) I and II only (c) II and III only (d) III only

(e) None are true

21. A dog who likes socks randomly selects two socks out of her master's sock drawer. The drawer contains 55 socks. Of the 55, 25 are black, 8 are white, 16 are navy, 2 are yellow, and 4 are tan. What is the probability that one black sock and one white sock are selected?
(a) $\frac{1}{2}$ (b) $\frac{55}{50}$ (c) $\frac{2 \cdot 25}{55^2}$ (d) $\frac{25+2}{55 \cdot 27}$ (e) $\frac{40}{27 \cdot 11}$

22. A scientist begins growing a culture of bacteria in the lab at 6 P.M. The initial bacterial population is 100 cells, and the population doubles every two hours. (The population of the bacteria grows exponentially.) When the population size reaches 1000 cells, the scientist must be back at the lab to harvest the colony. Of the following times, when is the latest the scientist can arrive and still be back in time.
(a) 9:30 P.M. (b) 1 A.M. (c) 9:30 A.M. (d) 12:30 A.M. (e) 12:45 A.M.

23. A teacher buys a box of doughnuts to share during office hours. Two students each eat twice as many as a third student. Then another three students come and eat two each. What is the smallest number of doughnuts the box could have contained if she purchased an odd number and all of the students ate at least one doughnut?
(a) 9 (b) 10 (c) 11 (d) 12 (e) 13

24. A typesetters apprentice, who is carrying a tray of letters forming the word MATHEMATICS, trips and spills the letters on the floor. If the apprentice randomly rearranges the letters into an eleven-letter word, what is the probability that the result will again be MATHEMATICS?
(a) $\frac{2!9!}{11!}$ (b) $\frac{2!2!2!}{11!}$ (c) $\frac{1}{8!}$ (d) $\frac{3!5!}{8!}$ (e) $\frac{1}{11!}$

25. $\overline{TP}$ is tangent to circle ABT. If $TP = 6$ and $PA = 3$, what is AB?

(a) 12 (b) 9 (c) 6 (d) 3 (e) Cannot be determined from the given information

26. Let $T(x) = \sin(x/2)\cos(x/3)$. The period of function T is

(a) 2π (b) 4π (c) 6π (d) 8π (e) 12π

27. Find and simplify the product $(x-1)(x^3+2x^2+3x+4)$.

(a) $x^4+x^3+5x^2+7x+4$ (b) $x^4+x^3+x^2+x-4$ (c) $x^4+x^3+x^2+x+4$

(d) $x^4+x^3+x^2+x+1$ (e) $x^4+x^3+x^2+x-1$

28. Which region or combination of regions in this Venn diagram represents the set $(A \cup B) - (B \cap C)$?

(a) V, VI (b) V (c) VII (d) I, II, III, IV (e) I, I, III

29. Evaluate $\lim_{x\to 4} \dfrac{x^3-4x^2-x+4}{x^2-16}$.

(a) $\dfrac{15}{8}$ (b) $-\dfrac{1}{4}$ (c) 1 (d) 0 (e) Does not exist

30. Find the domain of $f(x) = \dfrac{\log(x^2+5x+6)}{\lfloor x \rfloor - 1}$ where $\lfloor x \rfloor$ is the greatest integer function.

NOTE: The greatest integer function returns the value of the greatest integer less than or equal to x.

(a) $(-\infty, 1) \cup [2, \infty)$ (b) $(-\infty, -3) \cup (-2, \infty)$ (c) $(-\infty, -3) \cup [-2, \infty)$

(d) $(-\infty, 1) \cup (-2, \infty)$ (e) None of the above

31. The graph of the equation $\left(x + \dfrac{\sqrt{2}}{2}\right) y = \dfrac{x\left(x^2 - \dfrac{1}{2}\right)}{x - \dfrac{\sqrt{2}}{2}}$ intersects the unit circle $x^2 + y^2 = 1$ at how many different points?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

32. Find all points where $x^2 + y^2 - 6x + 6y = 18$ has a horizontal tangent.

(a) $(3, 3)$ and $(3, -9)$ (b) $(9, 3)$ and $(-3, 3)$ (c) $(-9, 3)$ and $(-3, 3)$

(d) $(3, -3)$ and $(3, -9)$ (e) None of the above

33. In right triangle ABC, $AB = 4$ and $BC = 3$. Determine $\sin(\angle B)$.

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{5}{4}$ (d) $\frac{\sqrt{11}}{4}$ (e) $\frac{\sqrt{7}}{4}$

34. Find the area bounded by the curves $y = 4x$, $y = 0$, and $x = 5$.

(a) 20 (b) 5 (c) 25 (d) 75 (e) 50

35. A drug is known to be only 60% effective in curing a certain disease. If eight patients with the disease are given the drug, find the (approximate) probability that at least one lives.

(a) 0.6 (b) 0.4 (c) 0.24 (d) 0.86 (e) 0.98

36. Evaluate $\lim_{x \to 1} \left(\frac{1}{x - 1} - \frac{2}{x^2 - 1} \right)$.

(a) Does not exist (b) Undefined (c) 0.5 (d) 0 (e) None of the above

37. In a survey 200 students were asked about their recycling habits.

55 students recycle paper

65 students recycle metal

75 students recycle glass

15 students recycle paper and metal

20 students recycle paper and glass

25 students recycle metal and glass

5 students recycle all three items

Find the number of students who recycled only paper.

(a) 55 (b) 25 (c) 20 (d) 15 (e) 5

38. What is the derivative of $y = 2e^{\pi} + 7$?

(a) 2 (b) $2e^{\pi}$ (c) $2\pi e^{\pi-1}$ (d) 0 (e) $2\pi e$

39. Two positive integers are chosen randomly. What is the probability that their product is even?

(a) 0 (b) 1 (c) 3/4 (d) 1/2 (e) 1/4

40. How many integers are contained in the solution set of the following inequality?

$$|3 - 4x| < 5$$

(a) 0 (b) 1 (c) 2 (d) 3 (e) Infinitely many

41. Given that $\tan q = b$, find $\tan(q + \frac{17\pi}{2})$.

(a) b (b) $-b$ (c) $\dfrac{1}{b}$ (d) $-\dfrac{1}{b}$ (e) Undefined

42. Simplify

$$\frac{2n^{1/3} - 4n^{-2/3}}{2n^{-2/3}}$$

(a) $n - 2$ (b) $n^{1/3} - 2$ (c) $n + 2$ (d) $\frac{2}{n}$ (e) $n^{2/3} - 2$

43. Find the exact value of

$$(\log_2 3)(\log_3 4)(\log_4 5) \cdots (\log_{31} 32)$$

(a) $\log_{31!}(3 \cdot 4 \cdot 5 \cdots 32)$ (b) $\ln(3 \cdot 4 \cdot 5 \cdots 32)$ (c) 5 (d) 16 (e) Undefined

44. Which statement is logically equivalent to the following statement?

If you're happy and you know it, you clap your hands.

(a) If you're not happy and you know it, you don't clap your hands.

(b) If you don't clap your hands, then you aren't happy or you don't know it.

(c) If you clap your hands, you're happy and you know it.

(d) If you're happy and you don't know it, you don't clap your hands.

(e) If you clap your hands and you aren't happy, then you don't know it.

45. A water tank is 6 feet long and has a cross section in the shape of a regular hexagon with side length 2 feet. The tank is lying on level ground with one side of the hexagon flat against the ground. If the depth of the water in the tank is two-thirds of the distance from the bottom to the top, what is the volume of water in the tank (rounded to two decimal places)?

(a) 62.35 cubic feet (b) 41.57 cubic feet (c) 42.67 cubic feet (d) 43.88 cubic feet (e) 50.27 cubic feet