1. Let x be an integer and suppose the mean and median of $x, 1, 4, 7$ and 12 are equal. What is the sum of the possible values of x?

   (A) 6

   (B) 8

   (C) 11

   (D) 13

   (E) 17

2. The area of the square inscribed in a semicircle is to the area of the square inscribed in the entire circle as:

   (A) 1:2

   (B) 2:3

   (C) 2:5

   (D) 3:4

   (E) 3:5

3. If $a + b = 4$ and $a^2 + b^2 = 9$, then what is $a^3 + b^3$?

   (A) 22

   (B) 16

   (C) 64

   (D) 28

   (E) 36

4. When a store sells a particular television for \$500.00, 20% of the revenue is profit. However, people are not buying the television! So they mark up the television to \$625.00 and then they put the newly priced television on sale, discounting it by 15%. With these adjustments, what percentage of the revenue is profit, to the nearest whole number?

   (A) 4%

   (B) 18%

   (C) 20%

   (D) 21%

   (E) 25%

5. What is the sum of the infinite series $0.3 + 0.3^2 + 0.3^3 + 0.3^4 + \cdots$?

(A) $\frac{107}{250}$

(B) $\frac{3}{11}$

(C) $\frac{3}{7}$

(D) $\frac{857}{2000}$

(E) $\frac{43}{100}$

6. If n is a positive integer, then powers of 2^n always end in one of the digits 2, 4, 6, or 8. There are only 4 digits in this last digit set. For $a \geq 2$ an integer, consider the set of last digits of a^n where n is a positive integer. What is the smallest possible size of such a set? What is the largest possible size of such a set?

(A) smallest = 1, largest = 4

(B) smallest = 1, largest = 5

(C) smallest = 2, largest = 9

(D) smallest = 1, largest = 9

(E) smallest = 2, largest = 5

7. If $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$, then z equals:

(A) $y - x$

(B) $x - y$

(C) $\frac{y-x}{xy}$

(D) $\frac{xy}{y-x}$

(E) $\frac{xy}{x-y}$

8. The population of a certain country grows by one person every 15 seconds. What is the rate of change of that country's population in people per year? (Assume there are 365 days in a year.)

(A) 3.1710×10^{-8}

(B) 4.7565×10^{-7}

(C) 6.6667×10^{-2}

(D) 2.1024×10^{6}

(E) 3.1536×10^{7}

9. What is the remainder when $x^{2014} + x^4 - x^2 + 1$ is divided by $x^2 + 1$?

   (A) 2

   (B) 0

   (C) 2011

   (D) 1

   (E) 12

10. Suppose $f(x) = (2x+3)^{n-2}$. If $f'(x) = 6(2x+3)^{n-3}$ then what is the value of n?

   (A) 3

   (B) 4

   (C) 5

   (D) 6

   (E) 8

11. The *power set* of a set A is the set of all subsets of A. Find the power set of $A = \{a, b, \{c, d\}\}$.

   (A) $\{a, b, \{c, d\}\}$

   (B) $\{a, b, c, d\}$

   (C) $\{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, b\}, \{b, c\}\}$

   (D) $\{\{\}, \{a\}, \{b\}, \{c, d\}, \{a, b\}, \{b, \{c, d\}\}, \{a, \{c, d\}\}, \{a, b, \{c, d\}\}\}$

   (E) $\{\{\}, \{a\}, \{b\}, \{\{c, d\}\}, \{a, b\}, \{b, \{c, d\}\}, \{a, \{c, d\}\}, \{a, b, \{c, d\}\}\}$

12. A straight line joins the points $(-1, 1)$ and $(3, 9)$. Its x-intercept is:

   (A) $\frac{-3}{2}$

   (B) $\frac{-2}{3}$

   (C) $\frac{2}{5}$

   (D) 2

   (E) 3

13. If $4^x - 4^{x-1} = 24$, then $(2x)^x$ equals:

   (A) $5\sqrt{5}$

   (B) $\sqrt{5}$

   (C) $25\sqrt{5}$

   (D) 125

   (E) 25

14. Suppose that $g_1, g_2, g_3, \ldots$ is a sequence defined as follows:

$$g_1 = 3, \quad g_2 = 5$$
$$g_k = 3g_{k-1} - 2g_{k-2} \text{ for all integers } k \geq 3$$

Find g_{20}

(A) 524287

(B) 104857

(C) 524289

(D) 1048577

(E) 2097153

15. Find the maximum value of the function

$$f(x, y, u, v) = \frac{x^2 + y^2 + 2xy}{u^2 + 2xy + y^2 + 2uv + v^2 + x^2}$$

(A) 0.9

(B) 1

(C) 1.2

(D) 2

(E) 1.5

16. A 3×3 magic square uses integers $1, 2, \ldots, 9$ once each in such a way that each column, each row, and each diagonal sums to 15. Find the value of N for the magic square below, only a portion of which is shown.

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

8		
N		7

17. A cylinder's radius is increased by 45% and its height is decreased by 20%. How has its volume changed?

(A) Increased by 36.4%

(B) Increased by 16.0%

(C) Increased by 68.2%

(D) Decreased by 36.7%

(E) Decreased by 64.5%

18. A security camera in an art gallery is mounted on a vertical wall 11 feet above the floor. What angle of depression (rounded to the nearest hundredth) should be used if the camera is to be directed at the center of a statue, given that the statue's center is located exactly 5 feet above the floor and 17 feet from the wall?

(A) $19.44°$

(B) $33.93°$

(C) $45°$

(D) $57.09°$

(E) $70.56°$

19. Find all vertical asymptotes of the graph of the function $f(x) = \dfrac{x^2 - 4}{x^2 - x - 2}$.

(A) $x = -1$

(B) $x = -1$ and $x = 2$

(C) $x = -1$ and $x = -2$

(D) $x = -1$, $x = -2$, and $x = 2$

(E) There are no vertical asymptotes.

20. Solve for x: $\log_2(x - 1) + \log_2(x + 1) = 3$.

(A) $x = 2$ only

(B) $x = -2$ and $x = 2$

(C) $x = 3$ only

(D) $x = -3$ only

(E) $x = -3$ and $x = 3$

21. Emmy has two sons: Bert and David. Emmy's age is ten more than twice Bert's age. In 5 years, David's age will be one-fourth of Emmy's current age. Currently, Bert's age is three times David's age. What is the age difference between Bert and David?

(A) 5 years

(B) 10 years

(C) 15 years

(D) 16 years

(E) 18 years

22. Which statement is logically equivalent to the following statement?

    If you fall, then you break your arm or your leg.

    (A) If you do not break your leg, then either you break your arm or you do not fall.
    (B) If you do not break your arm or your leg, then you do not fall.
    (C) If you break your arm or your leg, then you fall.
    (D) If you break your arm and your leg, then you fall.
    (E) You fall, break your arm and break your leg.

23. Suppose θ is an angle such that $\sin(2\theta) = \cos(\theta)$. Then $\tan(\theta)$ is

    (A) $\frac{1}{\sqrt{3}}$ only
    (B) $-\frac{1}{\sqrt{3}}$ only
    (C) undefined only
    (D) $\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$ only
    (E) $\frac{1}{\sqrt{3}}$, $-\frac{1}{\sqrt{3}}$ or undefined

24. The corners of a rectangle in the plane are located at the points (1,1), (1,10), (5,10) and (5,1). The line $y = mx$ divides the rectangle into two trapezoids of equal area. Find m.

    (A) $\frac{11}{6}$
    (B) 2
    (C) $\frac{13}{6}$
    (D) $\frac{10}{3}$
    (E) 3

25. A committee composed of Alice, Mark, Ben, Connie, and Francisco is about to select three representatives randomly. What is the probability that Connie is excluded from the selection?

    (A) 4/5
    (B) 3/5
    (C) 2/5
    (D) 1/5
    (E) 0

26. Suppose you are working in a new number system in which $i^2 = -1$, $j^2 = -1$, $k^2 = -1$, $i \cdot j \cdot k = -1$, and $k \cdot j \cdot i = 1$. Find the value of $i \cdot k$.

(A) i

(B) -i

(C) j

(D) -j

(E) k

27. A prisoner is given only 2 choices. Only one of those choices requires subchoices, of which there are 3. Only one of *those* subchoices requires further subchoices, of which there are 4. Only one of *those* further subchoices requires final subchoices, of which there are 5. Ultimately, how many options does the prisoner have?

(A) 5

(B) 11

(C) 14

(D) 24

(E) 120

28. Two swimmers start swimming at the same time at opposite ends of a 100 meter pool. One swimmer swims twice as fast as the other. How many times do they meet or pass each other by the time the fastest swimmer swims 600 meters?

(A) 3

(B) 4

(C) 5

(D) 6

(E) They never cross.

29. Let f be the function on the natural numbers defined by $f(x) = \begin{cases} \lfloor x^{\frac{1}{2}} \rfloor & \text{if} \quad x \text{ is even.} \\ \lfloor x^{\frac{3}{2}} \rfloor & \text{if} \quad x \text{ is odd.} \end{cases}$ Here $\lfloor x \rfloor$ is the greatest integer function. Find the smallest k such that $f^k(100) = 1$. Recall that $f^1(x) = f(x)$, $f^2(x) = f(f(x))$, $f^3(x) = f(f(f(x)))$ and so on.

(A) 3

(B) 5

(C) 7

(D) 8

(E) 10

30. Three landmarks of baseball achievement are Ty Cobb's batting average of .420 in 1911, Ted Williams's .406 in 1941, and George Brett's .390 in 1980. Here are the means and standard deviations of individual batting averages for all professional baseball players for these years: (1) 1911, mean = .266, std. dev. = .0371; (2) 1941, mean = .267, std. dev. = .0326; and (3) 1980, mean = .261, std. dev. = .0317. Which man performed better relative to his peers?

(A) Ty Cobb

(B) Ted Williams

(C) George Brett

(D) Ty Cobb and George Brett equally with respect to their peers

(E) none of the above

31. The six edges of a tetrahedron with vertices A, B, C and D measure 17, 23, 37, 67, 89 and 101 units. If edge AB is 101 units in length, what is the length of edge CD?

(A) 17

(B) 23

(C) 37

(D) 67

(E) 89

32. The sum of two numbers is 10; their product is 20. The sum of their reciprocals is:

(A) 1/10

(B) 1/2

(C) 1

(D) 2

(E) 4

33. There are five cowboys in a saloon. At high noon, each cowboy randomly chooses one of the other four cowboys and shoots him. What is the probability that exactly two of the cowboys are shot?

(A) 1/4

(B) 2/5

(C) 5/64

(D) 1/128

(E) 1/1024

34. When water changes into ice its volume increases by one ninth. Consider a 20-cm tall cylindrical beaker completely filled with ice. On top of the beaker, there is a cone of ice that has the same diameter as the beaker and is 5-cm tall. Assuming the beaker catches the water when the ice melts, what is the height of the water in the beaker when all of the ice melts? Round your answer to the nearest tenth.

(A) 2.4 cm

(B) 7.5 cm

(C) 19.3 cm

(D) 19.5 cm

(E) The beaker will overflow.

35. Students were gathered to recycle the following items: paper, metal, and glass.
165 students recycle paper
195 students recycle metal
225 students recycle glass
30 students recycle paper and metal
45 students recycle paper and glass
60 students recycle metal and glass
15 students recycle all three items

How many students recycled paper and nothing else?

(A) 75

(B) 165

(C) 105

(D) 140

(E) 120

36. The population of Flag City, Georgia was $82,800$ in 1980 and $98,600$ in 1998. If the growth of the population is exponential, what should be the approximate population of Flag City in the year 2010?

(A) $115,00$

(B) $109,134$

(C) $99,193$

(D) $110,775$

(E) $118,770$

37. An urn contains 10 marbles (6 red, 4 blue). Two marbles are chosen without replacement. Find the probability that at least one is red.

(A) $p = \dfrac{21}{25}$

(B) $p = \dfrac{8}{15}$

(C) $p = \dfrac{12}{25}$

(D) $p = \dfrac{13}{15}$

(E) $p = \dfrac{3}{5}$

38. Suppose $f(x) = \dfrac{1}{x}$. When $h \neq 0$, $\frac{f(x+h)-f(x)}{h}$ equals

(A) $\dfrac{1}{x(x+h)}$

(B) $\dfrac{-1}{x(x+h)}$

(C) $\dfrac{1}{x+h}$

(D) $\dfrac{-1}{x+h}$

(E) $\dfrac{1}{x}$

39. Find all exact solutions to $\dfrac{1}{2}\sin 2x - \sin^2 x = 0$ in the interval $-\dfrac{\pi}{3} \leq x \leq \dfrac{\pi}{3}$.

(A) $x = 0$ or $x = \dfrac{\pi}{4}$

(B) $x = \dfrac{-\pi}{4}$ or $x = \dfrac{\pi}{4}$

(C) $x = 0$ or $x = \dfrac{-\pi}{4}$

(D) $x = \dfrac{\pi}{4}$

(E) $x = 0$

40. The domain of $f(x) = \frac{\sqrt{3x+12}}{x^2-4}$ is

(A) $[-4,-2) \cup (-2,2) \cup (2,\infty)$

(B) $(\infty,-2) \cup (-2,2) \cup (2,\infty)$

(C) $(\infty,-2) \cup (-2,2) \cup [4,\infty)$

(D) $(-\infty,-4) \cup (-4,-2) \cup (-2,2) \cup (2,4) \cup (4,\infty)$

(E) none of the above

41. Consider the set $P = \{22, 6, 2, 6, 4\}$. The sum of the range, mean, median and mode is:

(A) 32

(B) 24

(C) 34

(D) 40

(E) 22

42. Let $M = \left(\begin{smallmatrix} 10 & 4 \\ 3 & 2 \end{smallmatrix}\right)$ and $N = \left(\begin{smallmatrix} 1 & a \\ 2 & 0 \end{smallmatrix}\right)$. Then MN equals

(A) $\left(\begin{smallmatrix} 10+4a & 20 \\ 3+2a & 6 \end{smallmatrix}\right)$

(B) $\left(\begin{smallmatrix} 18 & 7 \\ 10a & 3a \end{smallmatrix}\right)$

(C) $\left(\begin{smallmatrix} 10 & 4a \\ 6 & 0 \end{smallmatrix}\right)$

(D) $\left(\begin{smallmatrix} 10+3a & 4+2a \\ 20 & 8 \end{smallmatrix}\right)$

(E) $\left(\begin{smallmatrix} 18 & 10a \\ 7 & 3a \end{smallmatrix}\right)$

43. Given that line AC is parallel to line BD, $PA = 6$, $AB = 2$, and $AC = 4$, what is BD?

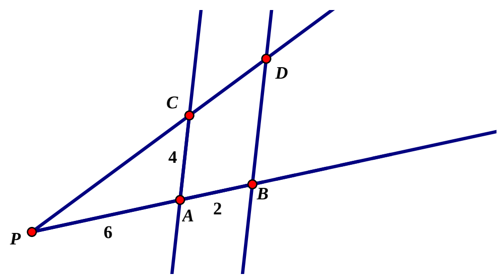
(A) $\frac{4}{3}$

(B) $\frac{3}{2}$

(C) 3

(D) $\frac{16}{3}$

(E) 12



44. The value of the infinite product $\frac{2}{11} \times \frac{5}{14} \times \frac{8}{17} \times \cdots \times \frac{3m-1}{3m+8} \times \cdots$ is

(A) ∞

(B) 0

(C) 0.03056

(D) 1

(E) 80

45. The set of points that satisfy the equation $x^2 - 6x = y^2 + 4y + 20$ all lie on

(A) an ellipse.

(B) a circle.

(C) a hyperbola.

(D) a pair of lines.

(E) a parabola whose axis is along the line $y = \frac{-3x}{2}$.